Scheme for the implementation of optimal cloning of arbitrary single particle atomic state into two photonic states

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We present a feasible scheme to implement the $1 \rightarrow 2$ optimal cloning of arbitrary single particle atomic state into two photonic states, which is important for applications in long distance quantum communication. Our scheme also realizes the tele-NOT gate of one atom to the distant atom trapped in another cavity. The scheme is based on the adiabatic passage and the polarization measurement. It is robust against a number of practical noises such as the violation of the Lamb-Dicke condition, spontaneous emission and detection inefficiency.

PACS numbers: 03.67.Mn, 03.67.Hk, 42.50.Dv

One of the most fundamental differences between classical and quantum information is that while classical information can be copied perfectly, quantum information cannot. In particular, it follows from the no-cloning theorem[1] that one cannot create a perfect duplicate of an arbitrary state. Although perfect cloning is not allowed, it is, nevertheless, possible to construct approximate[2] or probabilistic cloning[3]machine. The approximate cloning machine transforms the arbitrary input states into imperfect copies with probability one. While in probabilistic cloning one always obtain a perfect copy with some probability less than one. The simplest cloning machine is the duplication of a qubit, as was considered in [2]. Many generalizations and variants have followed, such as $N \rightarrow M$ optimal universal cloning machine for qubits[4], or d-level systems[5], state-dependent cloning machine[6], phase-covariant cloning machine for equatorial qubits[7], cloning machine for continuous-variable[8] systems etc. $1 \rightarrow 2$ and $1 \rightarrow 3$ universal cloning machines are also extended to the asymmetric case[9]. The study of quantum cloning has increased our understanding of the properties of quantum information. It has been shown that quantum cloning has close connection to the assessment of security[10] in quantum cryptography. Applications of quantum cloning can also be found in many quantum information [11] and quantum computation tasks[12]. All these motivations have led to the rapid development in the theory studies of quantum cloning[13].

On the other hand, it is important to find a specific physical means to carry out a given cloning process. Several schemes for realization of different quantum cloning processes have been suggested with quantum optics[14], linear optics[15], cavity QED[16] and spin networks[17]. Recently there have been greatly progresses in experiment for demonstrating the various kinds of cloning machines[18, 19]. In Ref.[20] a scheme for continuous variable cloning of light into an atomic ensembles has been proposed. However, the inverse process is also important because the photonic state is more suitable for long-distance communication. In this paper, we will propose a scheme to implement the $1 \rightarrow 2$ optimal cloning of arbitrary single particle atomic state into two photonic states. Our scheme combines the cavity QED technology and simple linear optical elements. The cavity QED system [21] is a promising candidate for quantum information processing. In cavity QED, the atoms act as the stationary qubits and they are coupled via interaction with the cavity photons. Our scheme is a combination of the two advantages: atom acts as stationary qubit used only for memory, while photons play the role of flying qubits. Our scheme also realizes the tele-NOT gate of one atom to the distant atom trapped in another cavity.

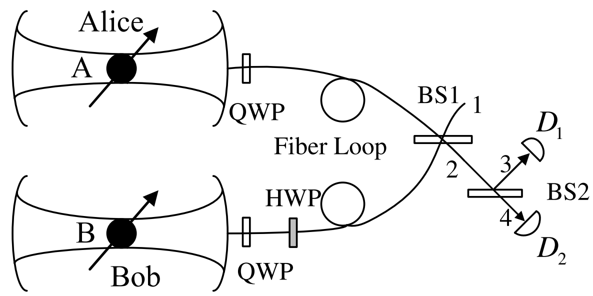

FIG. 1: Schematic setup to implement the $1 \rightarrow 2$ optimal symmetric cloning of arbitrary single particle atomic state into two photonic states. Here BS denotes beam splitter, QWP denotes quarter-wave plate and HWP is a 0° half wave plate.

Before presenting our scheme, let us review the $1 \rightarrow 2$ optimal cloning process firstly. Suppose the unknown single-qubit state to be cloned is in the form $|\psi\rangle_1 = a|0\rangle_1 + b|1\rangle_1$, where $|a|^2 + |b|^2 = 1$. The ancillary qubits are in a singlet state $|\psi\rangle_{23}^{-} = 2^{-\frac{1}{2}}(|01\rangle_{23} - |10\rangle_{23})$. The $1 \rightarrow 2$ optimal cloning of the state $|\psi\rangle_1$ corresponds to the projection operation onto the subspace of the total state $\Pi = |\psi\rangle_1 \otimes |\psi\rangle_{23}^{-}$, and the projector operator is given by[5]

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$$P_{123} = \left(\mathrm{I}_{12} - \left|\psi^{-}\right\rangle_{12}\left\langle\psi^{-}\right|_{12}\right) \otimes \mathrm{I}_3. \tag{1}$$

The procedure above will generate the state $\tilde{\Pi} = \sqrt{\frac{2}{3}}\left(|\zeta_1\rangle_{12}|1\rangle_3 - |\zeta_0\rangle_{12}|0\rangle_3\right)$, where $|\zeta_1\rangle_{12} = a|00\rangle_{12} + \frac{1}{2}b\left(|01\rangle_{12} + |10\rangle_{12}\right)$, and $|\zeta_0\rangle_{12} = b|11\rangle_{12} + \frac{1}{2}a\left(|01\rangle_{12} + |10\rangle_{12}\right)$. Here, the two cloning states emerge in 1 and 2 qubits. By tracing out the qubits 1 and 2, we find the qubit 3 realize optimal universal-NOT(UNOT) gate.

In the following we will give our scheme in detail. The system we are considering consists of two optical cavities with atoms A is trapped in Alice's cavity and the atom B in Bob's cavity as shown in Fig.1. The level structures of the atom A and B are depicted in Fig.2. Alice and Bob exploit two $F = 1$ hyperfine levels, while Bob exploits one additional hyperfine level. Such atomic level structures can be achieved in ^{87}Rb. The states $\left(5^2S_{\frac{1}{2}}, F = 1\right)$ and $\left(5^2P_{\frac{3}{2}}, F = 1\right)$ correspond to the $F = 1$ ground and excited hyperfine levels,respectively. The states $\left(5^2S_{\frac{1}{2}}, F = 2, m = 0\right)$ correspond to $|g_0'\rangle_B$. In our scheme, the atom A to be cloned is encoded in the superposition state of $|g_L\rangle$ and $|g_R\rangle$. With cavity A prepared in the vacuum state, the initial state of the whole system of Alice is

$$|\psi(0)\rangle_A = \left(a|g_L\rangle_A + b|g_R\rangle_A\right)|0,0\rangle_A, \tag{2}$$

where we have used the notation $|n_L,n_R\rangle_i$, $n_{L,R}$ represents the number of left- or right-circularly polarized photons and $i = A, B$ represents Alice or Bob, respectively. The transitions $|g_m\rangle_A \to |e_m\rangle_A\,(m = L, R)$ are driven adiabatically through the laser pulse collinear with the cavity axis, and atom A will be transferred with probability $P_1 \simeq 1$ to the state $|g_0\rangle_A$ by emitted a photon from the transitions $|e_L\rangle_A \to |g_0\rangle_A$ and $|e_R\rangle_A \to |g_0\rangle_A$. The corresponding Rabi frequency of the laser pulse is $\Omega_A(t)$; the transitions $|e_L\rangle_A \to |g_0\rangle_A$ and $|e_R\rangle_A \to |g_0\rangle_A$ are coupled to the left-circularly and right-circularly polarized mode of the cavity with the coupling rate g_A. In the rotating frame, the Hamiltonian of Alice's system is given by [22]

$$\begin{aligned}H_A = &-\left(\Delta + i\tfrac{\gamma_A}{2}\right)\left(|e_L\rangle\langle e_L| + |e_R\rangle\langle e_R|\right)_A + [\Omega_A(t)\,|e_L\rangle\langle g_L| \\ &|e_R\rangle\langle g_R|)_A + g_A\left(a_L^A|e_L\rangle\langle g_0| + a_R^A|e_R\rangle\langle g_0|\right)_A + H.c.],\end{aligned} \tag{3}$$

where γ_A and $a_{L,R}^A$ denote the atomic spontaneous emission rate and the annihilation operator for the corresponding polarized mode of the cavity, respectively. The Hamiltonian has two orthogonal dark state$|D_1(t)\rangle_A = \cos\theta_A(t)|g_L\rangle_A|0,0\rangle_A - \sin\theta_A(t)|g_0\rangle_A|1,0\rangle_A$ and $|D_2(t)\rangle_A = \cos\theta_A(t)|g_R\rangle_A|0,0\rangle_A - \sin\theta_A(t)|g_0\rangle_A|0,1\rangle_A$ with $\cos\theta_A = \frac{g_A}{\sqrt{|g_A|^2+|\Omega_A|^2}}$ and $\sin\theta_A = \frac{\Omega_A(t)}{\sqrt{|g_A|^2+|\Omega_A|^2}}$. Under the adiabatic approximation, the initial state (2) evolve into the following state:

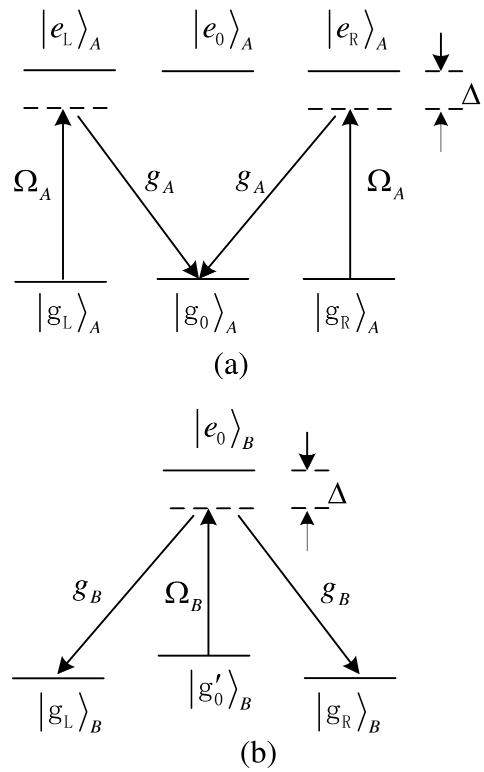

FIG. 2: The level configuration and transitions of the atom for Alice(a) and Bob (b).

$$\begin{aligned}|\Psi(t)\rangle_A &= \cos\theta_A(t)\left(a|g_L\rangle_A + b|g_R\rangle_A\right)|0,0\rangle_A \\ &- \sin\theta_A(t)|g_0\rangle_A\left(a|1,0\rangle_A + b|0,1\rangle_A\right).\end{aligned} \tag{4}$$

Alice slowly increasing Ω_A to satisfy the condition $\frac{g_A}{\Omega_A} \approx 0$ which will adiabatically transform the state $a|g_L\rangle_A + b|g_R\rangle_A$ into $a|1,0\rangle_A + b|0,1\rangle_A$.

The procedure for Bob is similar to Alice. Bob's system is prepared in the state $|g_0'\rangle_B|0,0\rangle_B$. The transitions $|g_0'\rangle_B \to |e_0\rangle_B$are driven adiabatically by laser pulse collinear with the cavity axis. $|e_0\rangle_B \to |g_L\rangle_B$ and $|e_0\rangle_B \to |g_R\rangle_B$ are coupled to the right-circularly and left-circularly polarized mode of the cavity with the coupling rate g_B.The atom of B will be transferred with probability $P_2 \simeq 1$ to the state $|g_L\rangle_B$ and $|g_R\rangle_B$ by emitted a photon from the transition $|e_0\rangle_B \to |g_L\rangle_B$ and $|e_0\rangle_B \to |g_R\rangle_B$. The Hamiltonian of Bob's system in the rotating frame is given by

$$\begin{aligned}H_B = &-\left(\Delta + i\tfrac{\gamma_B}{2}\right)\left(|e_0\rangle\langle e_0|\right)_B + [\Omega_B(t)\left(|e_0\rangle\langle g_0'|\right)_B \\ &+ g_B\left(a_R^B|e_0\rangle\langle g_L| + a_L^B|e_0\rangle\langle g_R|\right)_B + H.c.],\end{aligned} \tag{5}$$

Under the adiabatic approximation, Bob's system will evolve into the following state:

$$|\Psi(t)\rangle_B = \cos\theta_B(t)\,|g_0'\rangle_B\,|0,0\rangle_B - \frac{\sin\theta_B(t)}{\sqrt{2}}\left(|g_L\rangle_B\,|0,1\rangle_B + |g_R\rangle_B\,|1,0\rangle_B\right). \quad (6)$$

where $\cos\theta_B = \frac{\sqrt{2}g_B}{\sqrt{2|g_B|^2+|\Omega_B|^2}}$ and $\sin\theta_B = \frac{\Omega_B(t)}{\sqrt{2|g_B|^2+|\Omega_B|^2}}$. Bob also increase Ω_B adiabatically to map his state into a maximally entangled state $\frac{1}{\sqrt{2}}\left(|g_L\rangle_B\,|0,1\rangle_B + |g_R\rangle_B\,|1,0\rangle_B\right)$.

Because the cavities are one-side leaky, each cavity emits a photon and interferes at the beam splitter 1. The quarter-wave plates transform left-polarized and right-polarized photons into horizontally and vertically polarized photons with the transformation $|1,0\rangle \rightarrow |H\rangle$ and $|0,1\rangle \rightarrow |V\rangle$, where we have ignored the vacuum modes due to their no contribution to the click of the photondetectors. $|H\rangle$ and $|V\rangle$ denote the horizontally and vertically polarized photons, respectively. After the photons passing the two QWP and HWP, the total state of Alice and Bob's system will evolves into

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\,|g_0\rangle_A\left(a\,|H\rangle_A + b\,|V\rangle_A\right)\left(|g_R\rangle_B\,|H\rangle_B - |g_L\rangle_B\,|V\rangle_B\right). \quad (7)$$

In order to realize the $1 \rightarrow 2$ optimal cloning of the single qubit state, the central task is to realize the projective measurement given by Eq.(1). As shown in Ref.[19],the projective measurement in the form of Eq.(1) can be realized by the superposition of two modes on the 50:50 beam splitter. It can be identified by the simultaneous clicking of the detectors D_1 and D_2. We only consider the mode 2 because the same effect is expected on mode 1, for simplicity, we only depict the setup in mode 2 which applies to the mode 1 also. The setup of our scheme is depicted in Fig.1. The term $|HH\rangle_{AB}, |HV\rangle_{AB}, |VH\rangle_{AB}, |VV\rangle_{AB}$ will undergo the following transformation if there is twofold coincidence of D_1 and D_2

$$\begin{array}{l} |HH\rangle_{AB} \rightarrow |HH\rangle_{34}, \\ |HV\rangle_{AB} \rightarrow \frac{1}{2}\left(|HV\rangle_{34} + |VH\rangle_{34}\right), \\ |VH\rangle_{AB} \rightarrow \frac{1}{2}\left(|HV\rangle_{34} + |VH\rangle_{34}\right), \\ |VV\rangle_{AB} \rightarrow |VV\rangle_{34}. \end{array} \quad (8)$$

The scheme above requires the twofold coincidence event as the indication that the projector P_{123} in the form of Eq.(1) has been performed. Once the two detectors click simultaneously, the total state will evolves into

$$\begin{array}{l} \sqrt{\frac{2}{3}}a\,|HH\rangle_{34}\,|g_R\rangle_B - \sqrt{\frac{1}{6}}a\left(|HV\rangle_{34} + |VH\rangle_{34}\right)|g_L\rangle_B \\ -\sqrt{\frac{2}{3}}b\,|VV\rangle_{34}\,|g_L\rangle_B + \sqrt{\frac{1}{6}}b\left(|HV\rangle_{34} + |VH\rangle_{34}\right)|g_R\rangle_B. \end{array} \quad (9)$$

The equation above shows that the optimal $1 \rightarrow 2$ symmetric cloning process has been implemented. This scheme also realizes the tele-NOT gate of atom A to atom B trapped in another cavity. By taking consideration the coincidence of the mode 1, the total success probability of obtaining the two-fold coincidence event is $\frac{1}{4}$.

Next we give a brief discussion on feasibility of our scheme. In the actual situation, each photon leaks from the cavity in the form of single-pulse due to the random nature of the emission. The two photons interfere maximally when the two pulse shapes overlap completely at the beam splitter. In the adiabatic limit, the single-photon pulse shape is given by [23]$f_i(t) = \sqrt{\kappa_i}\sin\theta_i(t)\exp\left(-\frac{\kappa_i}{2}\int_0^t \sin^2\theta_i(\tau)\,d\tau\right)$, where κ_i denotes the cavity decay rate for Alice or Bob. As shown in Ref.[24], the difference between the two pulse shapes can be small enough if we choose the appropriate driving pulse. During our discussion we have assumed the coupling rates are fixed. However, the coupling rates have a variation in time due to the thermal motion of the atom, thus lead to the change of the output of the pulse shape $f_i(t)$. The result of the numerical simulation[22] shows our scheme works beyond the restriction of the Lamb-Dicke condition and the fidelity is only affected slightly. By utilizing the adiabatic method, the atomic decay is highly suppressed in our scheme. Another problem is the influence of the imperfections of the single-photon detectors, e.g., inefficient detections and the dark counts. If photons leak out of the cavity, but are not detected, these processes simply decrease the success probability by a factor of η^2 (η: the detection efficiency of the single-photon detectors), but have no influence on the fidelity of our scheme. For real single-photon detectors the dark counts can be in the few-percent region [25]. However, the utility of the twofold coincidence in our scheme will greatly suppress the dark counts. As such, the effect of the dark counts only affects the scheme slightly and can be safely neglected.

By combining the cavity QED technology and simple linear optical elements, we have proposed a scheme for the implementation symmetric quantum cloning of arbitrary single particle atomic state into two photonic states. The feature of our proposal is that the state to be cloned is encoded in the atomic state which is more suitable for memory. While the clones are appearing in two photonic states, which is important for long distance quantum communication protocols. Furthermore, our proposal is robust against a number of practical noises such as the violation of the Lamb-Dicke condition, spontaneous emission and detection inefficiency.

This work was supported by the National NSF of China, the Fok Ying Tung Education Foundation, and the Chinese Academy of Sciences.

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